AQA

A-LEVEL
MATHEMATICS

7357/3 - PAPER 3

Mark scheme

7357
June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

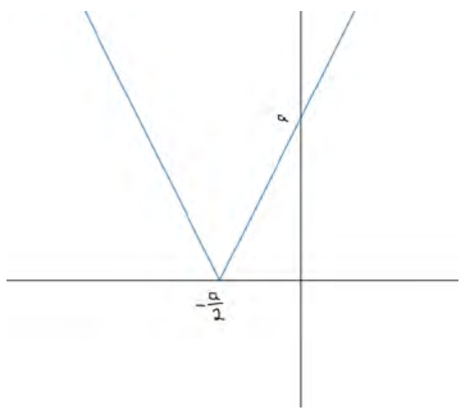
Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.1b	B1	9π
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer	AO1.1b	B1	7
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Circles correct answer	AO1.1b	B1	$3x-2y=7$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Draws the correct V-shape, nothing below x-axis	AO1.2	M1	
	Intersects negative x-axis with $-\dfrac{a}{2}$ labelled	AO1.1b	A1	
	Intersects positive y-axis with a labelled	AO1.1b	A1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
5	Uses small angle approximation for $\sin x$ or $\tan x$ Condone $y = 5 + 4x + 12x$ for this mark	AO1.1a	M1	$y = 5 + 4\sin\frac{x}{2} + 12\tan\frac{x}{3}$ $\sin x \approx x, \tan x \approx x$ $y \approx 5 + 4\left(\frac{x}{2}\right) + 12\left(\frac{x}{3}\right)$ $y \approx 6x + 5$ which is the equation of a straight line.
	Obtains correct equation Allow unsimplified form	AO1.1b	A1	
	Concludes that the graph can be approximated by a straight line. Requires simplification of equation (condone equals) and statement.	AO2.1	R1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Deduces that the lower bound of x is 1	AO2.2a	M1	$\{x \in \mathbb{R} : x > 1\}$
	States the domain in a correct form	AO2.5	A1	
6(b)	Differentiates using quotient rule (condone correct use of product rule) Must have $\mathrm{f}'(x) = \dfrac{(2x-2)^{\frac{1}{2}} - kx(2x-2)^{-\frac{1}{2}}}{(2x-2)}$ OE	AO1.1a	M1	$\mathrm{f}'(x) = \dfrac{(2x-2)^{\frac{1}{2}} - \frac{1}{2}x(2x-2)^{-\frac{1}{2}} \times 2}{(2x-2)}$ $= \dfrac{2x-2-x}{(2x-2)^{\frac{3}{2}}}$ $= \dfrac{x-2}{(2x-2)^{\frac{3}{2}}}$
	Obtains correct derivative in unsimplified form	AO1.1b	A1	
	Completes algebraic manipulation, with all previous working correct, to show the correct form. **AG**	AO2.1	R1	
6(c)	States that point of inflection requires second derivative to be 0	AO2.4	E1	For point of inflection $\mathrm{f}''(x) = 0$ $\mathrm{f}''(x) = \dfrac{(2x-2)^{\frac{3}{2}} - \frac{3}{2}(x-2)(2x-2)^{\frac{1}{2}} \times 2}{(2x-2)^3}$ $(2x-2)^{\frac{3}{2}} - 3(x-2)(2x-2)^{\frac{1}{2}} = 0$ $(2x-2)^{\frac{1}{2}}\left[(2x-2) - 3(x-2)\right] = 0$ $(2x-2)^{\frac{1}{2}}(4-x) = 0$ $x = 1$ or $x = 4$ $x \neq 1$ because of domain $\mathrm{f}''(3) = \dfrac{1}{32} > 0$ $\mathrm{f}''(5) = \dfrac{-\sqrt{2}}{256} < 0$ Therefore point of inflection at x=4
	Forms an equation $\mathrm{f}''(x) = 0$ OE	AO1.1a	M1	
	Solves their equation	AO1.1a	M1	
	Obtains solution x = 4	AO1.1b	A1	
	Gives a valid reason for rejecting x = 1, or cancels factor of $(2x-2)^{1/2}$ stating $x \neq 1$.	AO2.4	E1	
	Tests either side of 'their' x = 4	AO1.1a	M1	
	Completes rigorous argument to conclude they have one point of inflection Do not award this mark if 2nd E1 mark not awarded	AO2.1	R1	
6(d)	Deduces values of x for convex section of graph	AO2.2a	B1	$1 < x < 4$
	Total		**13**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)	Uses $n\log_a x = \log_a x^n$ correctly	AO1.1a	M1	$\log_a y = 2\log_a 7 + \log_a 4 + \frac{1}{2}$ $\Rightarrow \log_a y = \log_a 7^2 + \log_a 4 + \frac{1}{2}$ $= \log_a (49\times 4) + \frac{1}{2}$ $= \log_a 196 + \frac{1}{2}\log_a a$ $= \log_a 196 + \log_a \sqrt{a}$ $= \log_a 196\sqrt{a}$ $\therefore y = 196\sqrt{a}$
	Uses $\log_a x + \log_a y = \log_a xy$ or $\log_a x - \log_a y = \log_a \frac{x}{y}$ correctly	AO1.1a	M1	
	Obtains $\sqrt{a}$	AO1.1b	B1	
	Obtains correct answer in any correct form.	AO1.1b	A1	
7(b)	Explains that $-\frac{3}{2}$ should be rejected as it is not possible to evaluate $\log_a\left(-\frac{3}{2}\right)$	AO2.3	E1	$-\frac{3}{2}$ should be rejected as it is not possible to evaluate $\log_a\left(-\frac{3}{2}\right)$
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Recalls a correct trig identity, which could lead to a correct answer	AO1.2	B1	$(\text{LHS} \equiv)$ $\dfrac{\sin 2x}{1+\tan^2 x}$ $\equiv \dfrac{2\sin x\cos x}{1+\tan^2 x}$ $\equiv \dfrac{2\sin x\cos x}{\sec^2 x}$ $\equiv 2\sin x\cos x\cos^2 x$ $\equiv 2\sin x\cos^3 x$ $(\equiv \text{RHS})$
	Demonstrates a strategy for proving the identity, eg by converting all the terms on the LHS to cos and sin.	AO3.1a	M1	
	Concludes a rigorous mathematical argument to prove given identity **AG**	AO2.1	R1	
8(b)	Uses identity to write integrand in the form $a\sin 2\theta\cos^3 2\theta$	AO1.1a	M1	$\displaystyle\int \frac{4\sin 4\theta}{1+\tan^2 2\theta}\,\mathrm{d}\theta = \int 8\sin 2\theta\cos^3 2\theta\,\mathrm{d}\theta$ Let $u=\cos 2\theta$ then $\dfrac{\mathrm{d}u}{\mathrm{d}\theta}=-2\sin 2\theta \Rightarrow \sin 2\theta = -\dfrac{1}{2}\dfrac{\mathrm{d}u}{\mathrm{d}\theta}$ $I=-4\displaystyle\int u^3\frac{\mathrm{d}u}{\mathrm{d}\theta}\,\mathrm{d}\theta$ $=-4\displaystyle\int u^3\,\mathrm{d}u$ $=-u^4+c$ $=-\cos^4 2\theta+c$
	Correctly writes integrand as $8\sin 2\theta\cos^3 2\theta$	AO1.1b	A1	
	Selects an appropriate method for integrating, e.g. substitution $u=\cos 2\theta$, or by inspection PI by sight of $\cos^4 2\theta$	AO3.1a	M1	
	Obtains $k\displaystyle\int u^3\,\mathrm{d}u$ correctly PI by solution in form $k\cos^4 2\theta$, if by inspection	AO1.1a	M1	
	Obtains $-u^4$ or $-\cos^4 2\theta$ OE Only FT value of a	AO1.1b	A1F	
	Completes rigorous argument to obtain $-\cos^4 2\theta + c$ OE	AO2.1	R1	
	Total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)	Obtains correct length $\frac{w}{\sqrt{2}} = \frac{\sqrt{2}w}{2}$ ACF	AO1.1b	B1	$\frac{w}{\sqrt{2}}$
9(b)	Models the lengths as a geometric sequence	AO3.3	M1	$a = w$ and $r = \frac{1}{\sqrt{2}}$ $S_\infty = \frac{w}{1 - \frac{1}{\sqrt{2}}}$ $\approx 3.41w < 3.5w$
	Finds the sum to infinity provided their r < 1	AO1.1a	M1	
	Uses their model to obtain the correct sum in terms of w	AO3.4	A1	
	Compares their sum with 3.5w	AO2.4	E1	
9(c)	Explains that the model would have to include an additional 3 mm for each tile	AO3.5c	E1	The total length will now include an additional 3 mm for each tile. The total length will not have an upper limit.
	Explains that the total length will not have an upper limit Or The total length may now exceed 3.5w	AO3.5a	E1	
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
10	Begins proof by contradiction, assumes that $\sqrt[3]{2}$ is rational OE	AO3.1a	M1	Assume $\sqrt[3]{2}$ is rational
	Uses language and notation correctly to state initial assumptions	AO2.5	B1	$\sqrt[3]{2} = \frac{a}{b}$, a and b have no common factors
	Manipulates fraction including cubing.	AO1.1a	M1	$\Rightarrow \sqrt[3]{2}b = a$ $\Rightarrow 2b^3 = a^3$
	Deduces a is even	AO2.2a	R1	$\therefore$ a is even
	Deduces b is even	AO2.2a	R1	let $a = 2d$ then $2b^3 = 8d^3$ $\Rightarrow b^3 = 4d^3$ $\therefore$ b is even
	Explains why there is a contradiction	AO2.4	E1	Hence, a and b have a common factor of 2. This is a contradiction.
	Completes rigorous argument to show that $\sqrt[3]{2}$ is irrational	AO2.1	R1	$\therefore$ the assumption that $\sqrt[3]{2}$ is rational must be incorrect and it is proved that $\sqrt[3]{2}$ is an irrational number
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
11	Circles correct answer	AO1.1b	B1	$\frac{1}{10}$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
12	Circles correct answer	AO1.1b	B1	170 - 180
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
13	Explains that the actual recorded values for ‘Other takeaway food brought home’ are non-zero but have been rounded to the nearest whole number with reference to knowledge of the Large Data Set (could be implied) OE	AO2.4	E1	The values in the table are rounded to the nearest whole number so are actually non zero
	Explains that if unrounded numbers were used then the change could be calculated with reference to knowledge of the Large Data Set (could be implied) OE	AO2.4	E1	They are available to a large number of decimal places in the data set, which, if used, would show that the -29% is correct
	Deduces that Sarah’s claim is incorrect	AO2.2a	R1	Hence Sarah’s claim is incorrect
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(a)	Calculates $P(studies\ Physics) \times P(studies\ Geography)$ or Calculates $P(studies\ Geography \mid studies\ Physics)$ or $P(studies\ Physics \mid studies\ Geography)$	AO3.1b	M1	$P(P) \times P(G) = \frac{12}{24} \times \frac{8}{24} = \frac{1}{6}$
	Shows $P(studies\ Physics) \times P(studies\ Geography) = P(studies\ Physics \cap studies\ Geography)$ and correctly concludes that the events are independent or Shows that the appropriate conditional probability is equal to $P(studies\ Geography)$ or $P(studies\ Physics)$ and correctly concludes that the events are independent	AO2.1	R1	$P(P \cap G) = \frac{4}{24} = \frac{1}{6}$ Hence $P(P) \times P(G) = P(P \cap G)$ Therefore events are independent
14(b)	Uses conditional probability to calculate $P(M \cap B)$	AO3.1b	M1	$P(M \cap B) = P(M) \times P(B \mid M) = \frac{1}{5} \times \frac{3}{8} = \frac{3}{40}$
	Obtains the correct value of $P(M \cap B)$	AO1.1b	A1	$P(M) + P(B) - P(M \cap B) = \frac{1}{5} + \frac{1}{6} - \frac{3}{40} = \frac{7}{24}$
	Uses the addition rule to calculate $P(M \cup B)$	AO1.1a	M1	
	Obtains the correct value of $P(M \cup B)$	AO1.1b	A1	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
15(a)	States the correct binomial distribution	AO3.3	B1	B(6, 0.15)
15(b)	Calculates the correct probability	AO1.1b	B1	0.0000114
15(c)	Calculates $P(X \leq 1)$ or $P(X \leq 2)$ using the binomial distribution	AO1.1a	M1	$P(X \leq 1) = 0.7764$
	Obtains the correct answer	AO1.1b	A1	$P(X \geq 2) = 1 - P(X \leq 1)$ $P(X \geq 2) = 0.224$
15(d)	Finds the correct mean	AO1.1b	B1	0.9
15(e)	States a first appropriate assumption in context	AO3.5b	B1	The probability of a light bulb being faulty is fixed
	States a second appropriate assumption in context	AO3.5b	B1	A light bulb being faulty is independent of any other light bulb being faulty
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
16(a)(i)	Obtains the correct mean	AO1.1b	B1	1.38
16(a)(ii)	Uses the correct formula for standard deviation	AO1.1a	M1	$\sqrt{\frac{261.8}{120} - 1.38^2}$
	Obtains the correct standard deviation	AO1.1b	A1	0.526 to 0.529
16(b)(i)	Uses the model to calculate a normal probability	AO3.4	M1	0.5417 to 0.5428
	Obtains correct probability	AO1.1b	A1	
16(b)(ii)	Recalls correct value of 0	AO1.2	B1	0
16(c)	Calculates the value of mean – 3 x standard deviations	AO1.1b	M1	-0.1998 to -0.207
	Concludes that model might be inappropriate as the value is less than 0	AO2.2b	A1	This is less than 0 and so model might not be appropriate
16(d)	Standardises appropriately and formalises a probability statement PI by fully correct equation	AO3.1b	M1	$P\left(Z > \frac{0.75 - \mu}{0.21}\right) = 0.1$
	Obtains z value from inverse normal distribution (± 1.2816)	AO1.1a	M1	z value = 1.2816
	Forms a correct equation using standardised result and z value	AO1.1b	A1	$\frac{0.75 - \mu}{0.21} = 1.2816$
	Solves the equation to find the correct value of μ	AO1.1b	A1	μ =0.481
	Total		**12**	

Q	Marking Instructions	AO	Marks	Typical Solution
17 (a)	States both hypotheses correctly for two-tailed test	AO2.5	B1	X = number of matches won
	States model used **PI**	AO3.3	M1	H_0: p = 0.5
	Calculates $P(X \leq 6)$ or $P(X \leq 7)$ 0.828(1) or 0.945(3)	AO1.1a	M1	H_1: $p \neq 0.5$ Under null hypothesis X~B(10,0.5)
	Obtains the correct probability for $P(X \geq 7)$	AO1.1b	A1	
	Evaluates Binomial model by comparing $P(X \geq 7)$ with 0.05	AO3.5a	M1	$P(X \geq 7) = 1 - P(X \leq 6)$ $= 1 - 0.8281$ $= 0.172$
	Infers H_0 accepted CSO	AO2.2b	A1	
	Concludes correctly in context. (FT only available if previous M1 mark scored and B1 scored)	AO3.2a	E1F	$0.172 > 0.05$ Accept H_0 There is not sufficient evidence that Suzanne's new racket has made a difference
17(a)	**(ALTERNATIVE using critical region)** States both hypotheses correctly for two-tailed test	AO2.5	B1	X = number of matches won H_0: p=0.5
	States model used **PI**	AO3.3	M1	H_1: p≠0.5
	Considers critical region	AO1.1a	M1	
	Identifies critical region	AO1.1b	A1	Under null hypothesis X~B(10,0.5)
	Evaluates Binomial model by comparing $X = 7$ with critical values	AO3.5a	M1	
	Infers H_0 accepted CSO	AO2.2b	A1	$P(X \leq 1) = 0.0107 \text{ } or \text{ } 0.0108$
	Correctly concludes in context. 'Not sufficient evidence' or equivalent required.	AO3.2a	E1F	$P(X \geq 9) = 0.0107 \text{ } or \text{ } 0.0108$ Critical region is $X \leq 1$ and $X \geq 9$ $X = 7$ not in critical region Accept H_0 There is not sufficient evidence that Suzanne's new racket has made a difference

Q	Marking Instructions	AO	Marks	Typical Solution
17(b)	States model used **PI**	AO3.3	M1	Y~B(20,0.5)
	Expresses condition in terms of a cumulative probability statement PI by sight of $P(Y \geq c) < 0.1$	AO3.1b	M1	Require $P(Y > y) < 0.1$
	Tests one appropriate value for y	AO3.2b	R1	$P(Y \geq 13) = 0.1316 > 0.1$
	Obtains at least two correct cumulative probabilities	AO1.1b	A1	$P(Y \geq 14) = 0.0577 > 0.1$
	Obtains the correct minimum number of matches CSO	AO3.2a	A1	Minimum number of matches = 14
	Total		**12**	

	Marking Instructions	AO	Marks	Typical Solution
18(a)(i)	States opportunistic (sampling). Accept opportunity/convenience.	AO1.2	B1	Opportunistic sampling
18(a)(ii)	Explains that sample is not random.	AO3.5b	E1	The sample is not random.
18(b)	States both hypotheses correctly for one-tailed test	AO2.5	B1	H_0: μ = 66.5 H_1: μ < 66.5
	Formulates the test statistic	AO1.1a	M1	$z = \dfrac{65.4 - 66.5}{21.2/\sqrt{750}}$
	Obtains the correct value of the test statistic	AO1.1b	A1	$= -1.42$
	States the correct critical z-value OE	AO1.1b	B1	Critical z value = -1.28
	Infers H_0 rejected CSO	AO2.2b	A1	-1.42 < -1.28
	Correctly concludes in context. (FT only available if first B1 and M1 scored).	AO3.2a	E1F	Reject H_0 - there is sufficient evidence that the advertising campaign has reduced the consumption of chocolate.
	Total		**8**	

	TOTAL		**100**	